Uncovering Characteristic Quantities from Chaotic Time Series Distorted by Dynamical Noise

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Abstract

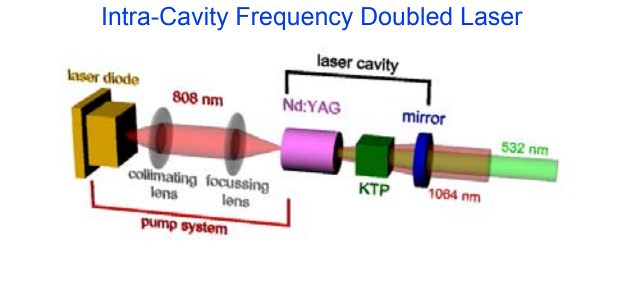
Noise in experimental time series plays a crucial role for the analysis of the data. In general one can distinguish between two fundamentally different types of noise in the data: measurement noise, which is passively added to the data during the measurement process, and dynamical noise, which interacts with the systems dynamics. In both cases results from time series analysis of the raw data are often questionable especially if one is interested in characterizing the underlying deterministic dynamics with chaotic signatures. Here we show the application of a method for extracting the deterministic dynamics from data distorted by dynamical noise. The underlying idea of the method is a Fokker-Planck-Analysis of the data. The investigations are performed from the viewpoint of an experimentalist, i.e., there is only one system variable or a linear combination of a only few variables measurable as it is usual for an experimental situation. We show the possibility of uncovering characteristic quantities like dimensions, Lyapunov exponents and power spectrum by the method. First, the method is demonstrated on numerically integrated data of a non-linear dynamical system perturbed by dynamical noise. In a second step we apply the method to data gained from a laser experiment with an unknown deterministic dynamics.

Motivation

Some nonlinear systems not only exhibit chaotic behavior they are also disturbed by intrinsic or extrinsic noise sources. This noise can act in different ways on the dynamics of the system. The analysis on the bases of nonlinear time series analysis of the system is at least hindered or even impossible.

The Experimental System

Intra-Cavity Frequency Doubled Laser

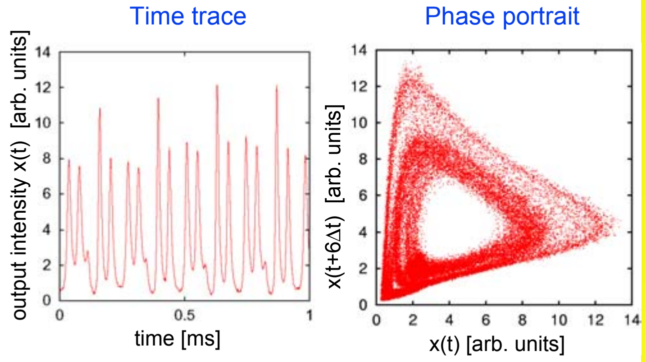


Intracavity frequency doubled solid state laser are of great technical interest because of their extraordinary features. They are easy to handle, reliable, compact, and very efficient. The laser consists of a laseractive crystal (e.g. Nd:YAG — neodymium-doped yttrium aluminum garnet), which is pumped typically by infrared laser diodes, and a optically nonlinear crystal (e.g. KTP — potassium titanyl phosphate), which converts the infrared laser light (wavelength 1064nm) to green light of 532nm wavelength.

The underlying physical process of frequency conversion, which relies on a highly nonlinear process within the KTP, is leading to a major obstacle for many technical applications:

The output of the laser output intensity fluctuates dramatically.

This is often called green problem in the literature.



Idea

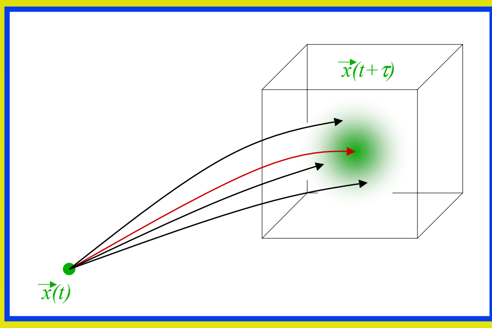
Reconstruction of the pure deterministic dynamic of the system and time series analysis on the reconstructed data.

The Method

The dynamics of the system subjected to dynamical noise can be described with a Langevin equation:

$$\dot{\vec{x}} = f(\vec{x},t) + g(\vec{x},t)\cdot\Gamma(t)$$

where f denotes the deterministic and g the diffusive part. This means the system develops similar to a deterministic system but the dynamics is blurred.



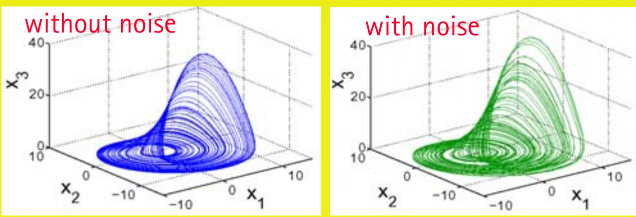
The different steps:

- search points in the ε-neighborhood in the reconstructed phase space of certain point x(t)
- follow each of the trajectories a time τ starting at the neighbors of x(t)
- the mean over the points x(t+τ) is interpreted as the deterministic part of the trajectory and the deviation from the mean as the noise part
- the vector pointing from x(t) to x(t+τ) represents the time evolution of the deterministic differential equation
- the deterministic part of the flow in the phase space can be reconstructed by covering the entire phase space with this method

Investigation of the Rössler System with Additional Noise

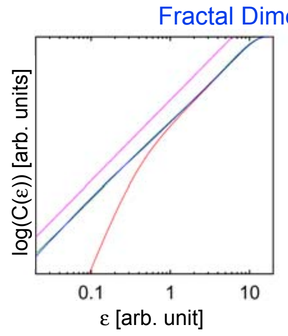
$$\dot{x}_1 = -x_2 - x_3$$
$$\dot{x}_2 = x_1 + 0{,}36x_2 + 0{,}2\Gamma(t)$$
$$\dot{x}_3 = 0{,}4 + x_3(x_1 - 4{,}5)$$

Γ denotes Gaussian white noise



Investigation of the Noisy Rössler System

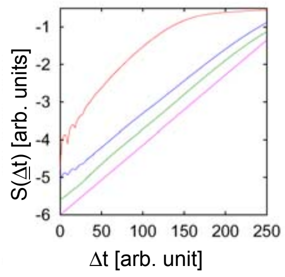
Fractal Dimension



In case of the deterministic, the noisy and reconstructed data

The line has the slope of the numerically determined dimension D_{KY}=2,04.

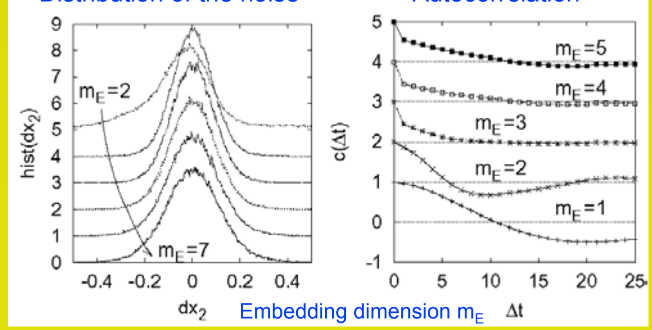
Lyapunov Exponent



In case of the deterministic, the noisy and reconstructed data

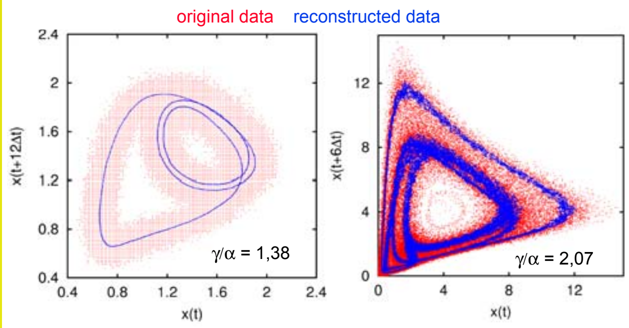
The line has the slope of the numerically determined Lyapunov exponent λ=0,148·h.

The Noise Part of the Rössler dynamics

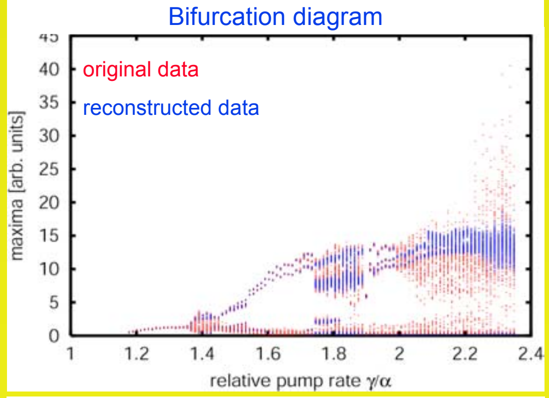


Application to the Data from the Laser

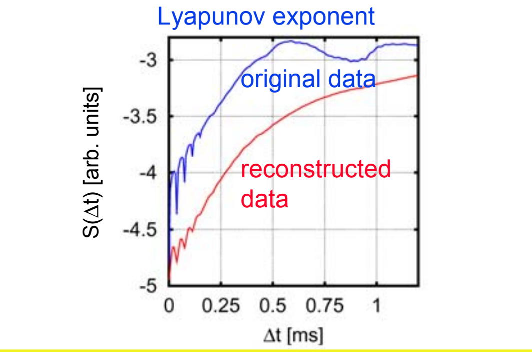
Phase portrait



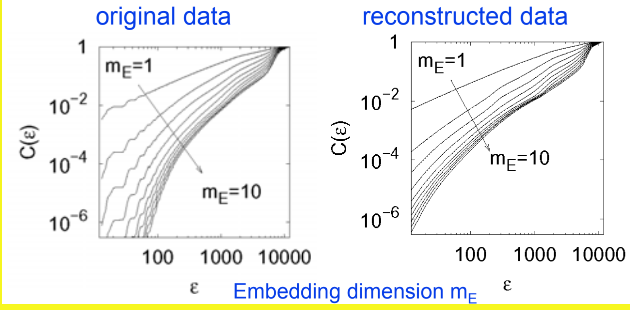
Bifurcation diagram



Lyapunov exponent



Dimension of the



Conclusion

- The deterministic part can be reconstructed from a system subjected to noise
- The characteristic quantities can be determined but are slightly underestimated
- The noise In the system can be characterized

Outlook

- Problems due to the embedding have to be solved (sufficient dimension, correlations)
- The limitations of the method have to be determined

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